ELECTROSTATICS

It is found that when amber is rubbed with cat skin attracts light objects then the amber is said to be charged. Similarly, two glass rods rubbed with silk cloth when brought together repel each other. Rubbed glass rod and silk cloth attract each other. This shows that there are two kinds of charges, the charge present on the glass rod is taken as positive and that on silk cloth is considered as positive.

Note: 1. The positive and negative charges are named by Benjamin Franklin.

2. Gold leaf electroscope is used to detect nature of charge.

3. When an atom acquires surplus of electrons, it become negatively charged. When an atom loses some electrons, it becomes positively charged. Atom as a whole is electrically neutral, because it contains equal number of protons and electrons.

4. Electrostatics deals with the study of electric charges at rest.

CHARGE: It is the basic property of matter by virtue which it repels its own kind but attracts the opposite kind. Or

The additional property of protons and electrons which give rise to electric force between them.

Charge of proton is + 1.6 X 10^{-19}C.

Charge of electron is - 1.6 X 10^{-19}C

Charge of neutron is Zero.

Properties of electric charge.

1. Like charges repel each other whereas unlike charges attract each other.
2. Charge is quantized.
3. Charge is conserved.
4. Charges is a scalar.
5. Charges of a body is independent its motion.
6. Charges resides only on the outer surface of the charged conductor.
7. Distribution of charges over a conductor depends on its shape.
8. A stationary charge produces an electric field only where as an accelerated produce both electric and magnetic field.

Charge is quantized.

If a body contains $n_1 e + n_2 (-e) = (n_1 - n_2) e$. Since n and n are integers, their difference is also an integer. Thus charge on a body is always integral multiple of e (q = ne) and can be increased or decreased in steps of e..

Charge is conserved.

In a isolated system, the total charge always remains constant. When we rub two bodies' one body gains charges and other body loses charges, but it is found that the total charge of the isolated system is always conserved. It is not possible to create or destroy net charge carried by any isolated system although the charge carrying particles may be created or destroyed in a process. In nature a neutron convert in to a proton and an electron. The proton and an electron thus created have equal and opposite charges, hence total charge is zero before and after creation.

Differences between charge and mass.

Charge	Mass
There are two types of charges	There is one type of mass
Force between charges is either attractive or repulsive.	Force between masses in only attractive.
Charge on a body is independent of its motion.	Mass of a body depends on its motion.
It is a quantized physical quantity.	It is not a quantized physical quantity.

Note :

1 A body can be charged by three methods. They are friction, conduction and induction.

2 Conductor can be charged by friction or induction.

3 Insulators can be charged by friction only.

4 When a conductor is charged, charges distributes over its outer surface.

5 When an insulator is charged, charged remains localized.

6 Repulsion is the sure test for charging of two bodies.

7 During induction, the conductor gets equal and opposite nature of charge.

8. In the case of spherical conductor, since curvature of surface is uniform, distribution of charges on it is also uniform.

9. Charge on a body is independent of its motion where as the mass of a body depends on its motion.

Charging by induction:

1 two metal spheres P and Q supported on insulating stands in contact as shown in **fig1.**

2 Bring positively charged rod near the sphere P and take care that it should not touch the sphere **fig2.**

3 Now free electrons in the spheres are attracted towards the charged rod. Hence left surface of sphere **P** has excess negative charge and rare surface of sphere Q acquire excess positive charge **fig3.**

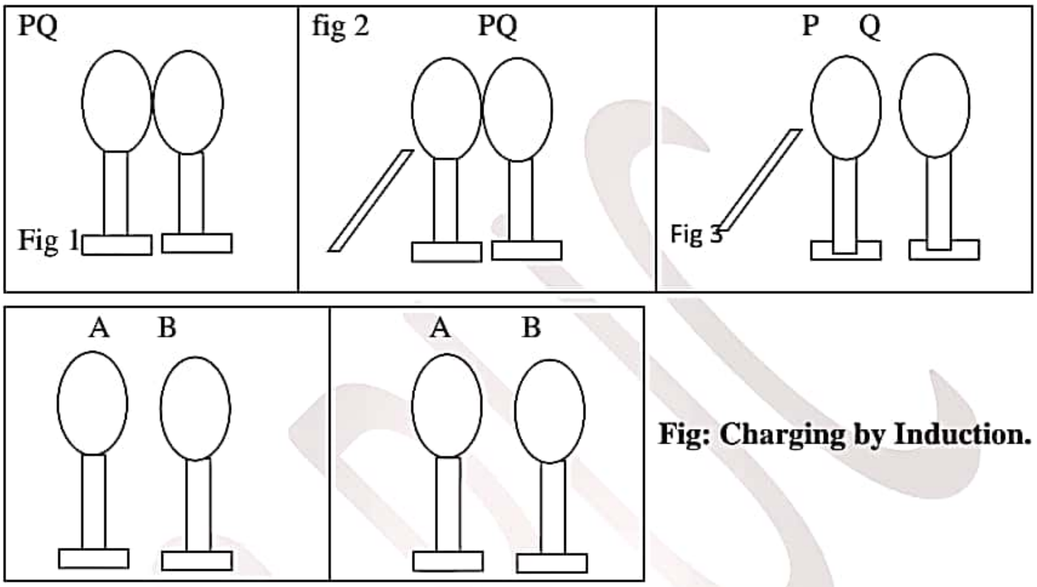


Fig: Charging by Induction.

4 These charges are called bound charges or accumulated charges.

5 These charges remains on the surface as long as the charged rod is held near the sphere.

6 If the rod is removed, the bound charges redistributed to their original neutral state.

7 Separate the spheres by a small distance **(fig.4)** with charged rod held near the sphere A. Two spheres are found oppositely charged and attract each other.

8 If we remove the rod, the charges in the spheres arrange themselves as in **fig 4.**

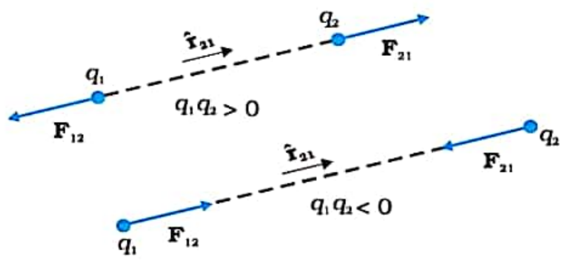
9 Now separate the spheres quite apart. It is found that charges on them get uniformly distributed over them as in **fig 5.**

10 In this process two metal spheres are equal and oppositely charged.

State and explain Coulomb's law in electrostatics

Statement: *Electrostatic force of attraction or repulsion between any two stationary point charges is directly proportional to product of magnitude of their charges and inversely proportional to square of the distance between them. This force acts along the line joining the centers two charges.*

Force is attractive if the charges are unlike and repulsive if the charges are like.



(b)

Explanation:

Consider two charges $\mathbf{q_1}$ and $\mathbf{q_2}$ separated by the distance **d**.

Then from coulomb's law $\mathbf{F \propto q_1\, q_2}$---- (1) and $\mathbf{F \propto 1/d^2}$---- (2)

From (1) and (2)

$$F \propto \frac{q_1\, q_2}{d^2}$$

$$F = K\frac{q_1\, q_2}{d^2}$$

Where K is the constant of proportionality Its value depends on

1 System of unit chosen (to measure F, q and d).
2 Nature of medium in which charges are placed.

In S.I system and for free space $K = \frac{1}{4\pi\epsilon_0} = 9\times10^9\ Nm^2\ C^{-2}$

Where ε_0 is the permittivity of free space. Its value is $8.854 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1\, q_2}{d^2}$$

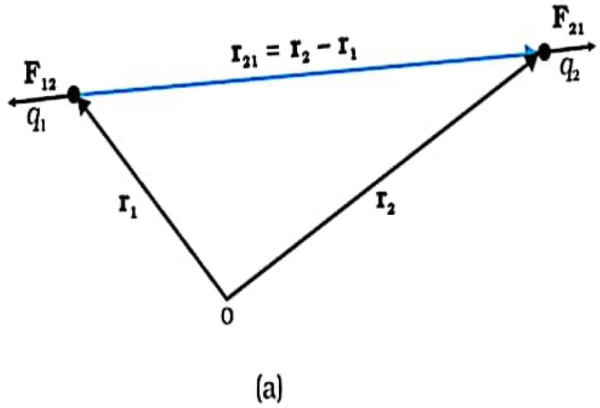
S.I unit of electric charge is **coulomb**

Definition of coulomb: Consider $F = \frac{1}{4\pi\epsilon_0}\frac{q_1\, q_2}{d^2}$

Let $q_1 = q_2 = \pm 1C$ then $F = 9X10^9N$

One coulomb is that charge which when placed at a distance of one meter from an equal and similar charge in air repels it with a force of $9X10^9N$.

Coulomb's law in vector form:



$\overrightarrow{F_{12}}$ is the force acting on q_1 due to q_2 $\overrightarrow{F_{21}}$ is the force q_2 due to q_1 and $\overrightarrow{r_{21}}$ is a vector acting from q_1 to q_2

$$\overrightarrow{r_{21}} = \overrightarrow{r_2} - \overrightarrow{r_1}$$

$\overrightarrow{r_{12}}$ is the vector acting from q_2 to q_1

$$\overrightarrow{r_{12}} = \overrightarrow{r_1} - \overrightarrow{r_2}$$

The magnitude of $\overrightarrow{r_{21}}$ is r_{21} and $\overrightarrow{r_{12}}$ is r_{12} also $r_{12} = r_{21}$ and $\overrightarrow{r_{12}} = -\overrightarrow{r_{21}}$

$\overrightarrow{r_{12}} = -r_{12}\widehat{r_{12}}$ or $\widehat{r_{12}} = \frac{\overrightarrow{r_{12}}}{r_{12}}$ also

$\overrightarrow{r_{21}} = r_{21}\widehat{r_{21}}$ or $\widehat{r_{21}} = \frac{\overrightarrow{r_{21}}}{r_{21}}$

Coulomb's force between two point charges q_1 and q_2 located at $\overrightarrow{r_1}$ and $\overrightarrow{r_2}$ and if the force exerted by q_1 on q_2 is

$$\overrightarrow{F_{12}} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{{r_{21}}^2}\widehat{r_{21}} \quad \text{or} \quad \overrightarrow{F_{21}} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{{r_{12}}^2}\widehat{r_{12}}$$

There fore, $\vec{F}_{12} = -\vec{F}_{21}$

Conclusion: 1. Coulomb's law agrees with Newton's third law of motion.

2. Like charges repel each other and unlike charges are attract each other.

Absolute and relative permittivity

Consider two charges $\mathbf{q_1}$ and $\mathbf{q_2}$ separated by distance **d** in air, then

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{d^2} \text{------- (1)}$$

Force between same two charges separated by same distance in medium is

$$F_{med} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{d^2} \text{-------- (2)}$$

Where εis the permittivity of medium.

Consider (1) / (2), then

$$\frac{F_a}{F_{med}} = \frac{\varepsilon}{\epsilon_0} = \varepsilon_r$$

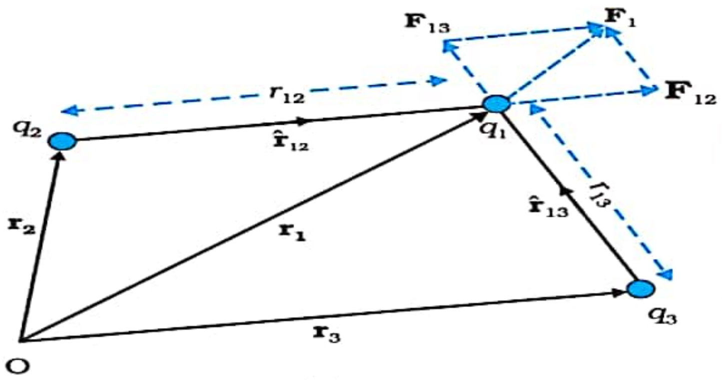
Where ε_r is the relative permittivity of dielectric constant of medium is defined as the ratio of force between two charges separated by certain distance in air to the force between same two charges separate by same distance is a medium. **or**

The ratio of the permittivity of the material medium to the permittivity of the free space or air.

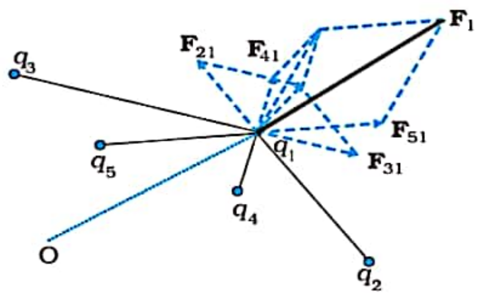
Force between multiple charges

Force between two point charges is given by Coulomb's law. But when there are several charges, Coulomb's law is insufficient to find the force on a given charge.

Experimentally it is verified that force on any charge due to number of other charges is the vector sum of all the forces on that charge due to the other charges taken one at a time. ***The individual forces are unaffected due to the presence of other charges. This is the principle of superposition.***



(a)



Consider the system of three charges $\mathbf{q_1}$, $\mathbf{q_2}$ and $\mathbf{q_3}$ as shown in the fig. Force on $\mathbf{q_1}$ due to $\mathbf{q_2}$ and $\mathbf{q_3}$ is the vector sum of forces due to each charges.

$\vec{F}_{12}$ is the force on $\mathbf{q_1}$ due to $\mathbf{q_2}$ and is given by $\vec{F}_{12} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2_{12}}\hat{r}_{12}$

$\vec{F}_{13}$ is the force on $\mathbf{q_1}$ due to $\mathbf{q_3}$ and is given by $\vec{F}_{13} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_3}{r^2_{13}}\hat{r}_{13}$

Total force on $\mathbf{q_1}$ due to $\mathbf{q_2}$ and $\mathbf{q_3}$ is $\vec{F} = \vec{F}_{12} + \vec{F}_{13} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2_{12}}\hat{r}_{12} + \frac{1}{4\pi\epsilon_0}\frac{q_1 q_3}{r^2_{13}}\hat{r}_{13}$

If the system contains **n** charges $\mathbf{q_1}$, $\mathbf{q_2}$, $\mathbf{q_3}$----- q_n then the net force on $\mathbf{q_1}$ due to other charges is

$$\vec{F} = \vec{F}_{12} + \vec{F}_{13} + \dots\dots\dots + \boldsymbol{F_{1n}} = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1 q_2}{r^2_{12}}\hat{r}_{12} + \frac{1}{4\pi\epsilon_0}\frac{q_1 q_3}{r^2_{13}}\hat{r}_{13} + \dots\dots\dots + \frac{q_1 q_n}{r^2_{1n}}\right]$$

$$= \frac{q_1}{4\pi\varepsilon_0} \sum_{i=2}^{n} \frac{q_i}{r^2_{1i}} \hat{r}_{1i}$$

The vector sum is obtained by parallelogram law of vector addition.

Electric field

Electric intensity at a point in an electric field is defined as the force experienced by unit positive charge placed at that point in a field.

Note:

- Electric field is the region where electric charge experience electric force.
- Strength of electric field is measured by the quantity called electric field strength or electric intensity.
- Electric intensity is a vector quantity.
- Direction of electric field is away from positive charge and towards the negative charge.

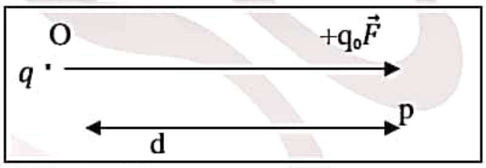
Significance of electric field:

It gives the magnitude and direction of force acting on a unit positive charge.

Expression for electric intensity at a point due to a point charge.

Consider a point charge **+q** at **O** in air. Let **P** be a point at a distance **d** from the charge **q**. By definition electric intensity at a point **P** is the force experience by unit positive charge placed at **P**. Consider a charge $\mathbf{q_0}$ at **P.**

From Coulomb's law



WKT $E = F/q_0$ ………… (1)

Now electric force acting on q(point charge) charge due to q_0 (test charge) is given by

$$\vec{F}\,(r) = \frac{1}{4\pi\epsilon_0} \frac{q\, q_0}{d^2} \hat{P}$$

Where $\hat{P}$ is the unit vector acting along the direction of force.

let substitute the value of F in eqn (1)

eqn (i) becomes,

Electric intensity $\vec{E}\,(r) = \frac{\vec{F}\,(r)}{q_0} = \frac{1}{4\pi\epsilon_0} \frac{q}{d^2} \hat{P}$

Electric dipole: It is a system of two equal and opposite charges separated by certain distance (2a). Eg: sodium chloride, molecules of water, alcohol etc.

Electric dipole moment : It is defined as the product of magnitude of one of the charges and distance of separation between the charges forming dipole.

- Strength of electric dipole is measured by quantity called electric dipole moment.
- Electric dipole moment p = q. 2a
- Electric dipole moment is a vector.
- Direction of electric dipole moment is from negative charge to the positive charge and is along the line joining the charges.
- S.I unit of electric dipole moment is coulomb – meter (C-m).
- Net charge of electric dipole is zero.

Uniform electric field: If the electric intensity at all points in an electric field is same in magnitude and direction, then the electric field is uniform.

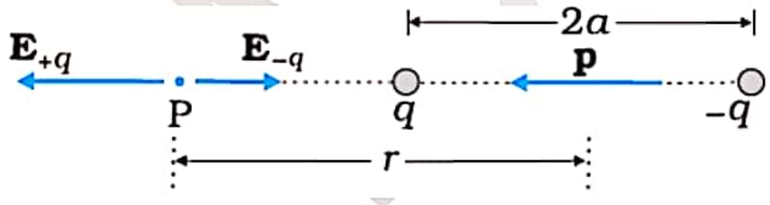
- Electric field between oppositely charged parallel plates separated by a small distance.
- Electric field produced by a thin and finite sheet of uniform charge density.

Non –uniform electric field: If the electric intensity at all points in an electric field is different in magnitude or in direction or both then the electric field is non-uniform.

- Electric field around a point charge.
- Electric field between oppositely charged parallel plates separated by a large distance.

Derive expression for electric field at a point on the axis of dipole.

Consider an electric dipole consisting of two charges +q and –q separated by distance 2a. Let P be a point at a distance r from the center of the dipole.



Electric field at P due to charge –q is $\overrightarrow{E_{-q}} = \frac{q}{4\pi\varepsilon_0(r+a)^2}\ (-\hat{P})$

Where P is the unit vector along the dipole axis (from –q to +q)

Electric field at P due to charge +q is $\overrightarrow{E_{+q}} = \frac{q}{4\pi\varepsilon_0 (r-a)^2}\hat{P}$

Total electric field at P is $\vec{E} = \vec{E}_{+q} + \vec{E}_{-q} = \frac{q}{4\pi\varepsilon_0 (r-a)^2}\hat{P} - \frac{q}{4\pi\varepsilon_0 (r+a)^2}\hat{P}$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]\hat{P}$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{(r+a)^2 - (r-a)^2}{(r-a)^2 (r-a)^2}\right]\hat{P}$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{r^2+a^2+2ra-(r^2+a^2-2ra)}{(r-a)^2 (r-a)^2}\right]\hat{P}$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{r^2+a^2+2ra-r^2-a^2+2ra)}{(r-a)^2\,(r+a)^2}\right]\hat{P}$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{4ra}{(r^2-a^2)^2}\right]\hat{P}$$

For r >>> a, a is neglected

$$= \frac{q}{4\pi\epsilon_0}\frac{4ra}{r^4}\hat{P}$$

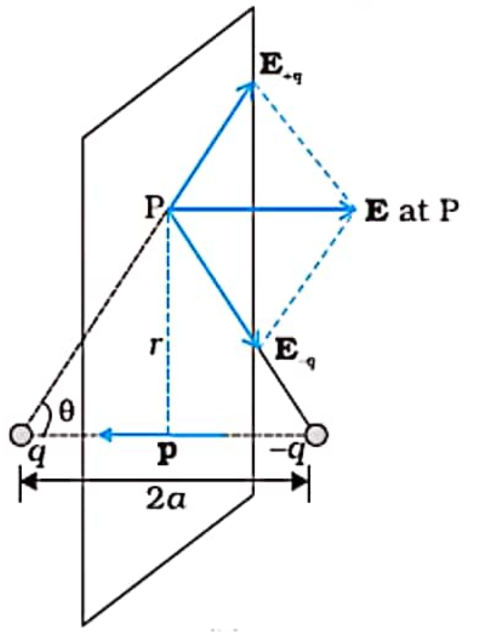
$$= \frac{q}{4\pi\epsilon_0}\frac{4a}{r^3}\hat{P}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{2\,2aq}{r^3}\hat{P}$$

$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{2P}{r^3}\hat{P}$ where 2a.q=P= Dipole moment of dipole.

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{2\vec{P}}{r^3}$$

For Points on the equatorial line :



Consider a dipole consisting of two charges **q** and**-q** separated by distance **2a**. Let **P** be a point o the equatorial line at a distance **r** from the centre of dipole.

Electric field at P due to charge +q, $\vec{E}_{+q} = \frac{q}{4\pi\epsilon_o}\frac{1}{(r^2+a^2)}$

Electric field at P due to charge –q, $\vec{E}_{-q} = \frac{q}{4\pi\epsilon_o}\frac{1}{(r^2+a^2)}$

The directions of $\vec{E}_{+q}$and $\vec{E}_{-q}$are shown in diagram.$\vec{E}_{+q}$ and $\vec{E}_{-q}$ can be resolve into two comonents.one component is to parallel to dipole axis and another component is to perpendicular to the dipole axis.The components perpendicular to the dipole axis cancel each other and the components parallel to dipole axix adds up. Hence resultant electric intensity at P is

$$\vec{E} = -(\vec{E}_{+q} + \vec{E}_{-q})\, \text{Cos}\theta \hat{P}$$

$$= -\left(\frac{q}{4\pi\epsilon_o}\frac{1}{(r^2+a^2)} + \frac{q}{4\pi\epsilon_o}\frac{1}{(r^2+a^2)}\right) \text{Cos}\theta\, \hat{P}$$

$$= -\frac{2q}{4\pi\epsilon_o}\frac{1}{(r^2+a^2)}\frac{a}{(r^2+a^2)^{1/2}}\hat{P}$$

$$= -\frac{2qa}{4\pi\epsilon_o}\frac{a}{(r^2+a^2)^{3/2}}\hat{P}$$

If r >>> a, then a is neglected

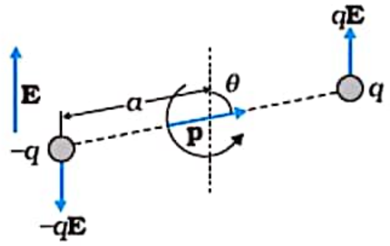
$$= -\frac{2qa}{4\pi\epsilon_o}\frac{1}{r^3}\hat{P}$$

$$\vec{E} = -\frac{1}{4\pi\epsilon_o}\frac{P}{r^3}\hat{P}$$

$$\vec{E} = -\frac{1}{4\pi\epsilon_o}\frac{\vec{P}}{r^3}$$

Negative sign shows that electric field is opposite to dipole axis. From equations (1) and (2) it is observed that electric field at a point on the axis is twice the field at a point on the equatorial line.

Torque on dipole in uniform electric field



Consider a dipole placed in uniform electric field of Intensity $\vec{E}$. Let θ is the angle between the dipole axis and Direction of electric field. The charge q experience force qE
In the direction of field. But the charge – q experience a force $-qE$ opposite to electric field. Force Since electric field is Net force on dipole is zero. But uniform, it experience torque.

Expression for magnitude of torque on dipole is
τ= magnitude of one of force x arm of couple

= q E x 2a sinθ
= q 2a E Sinθ where q = magnitude of one of the charges of dipole.
P = q.2a = Dipole moment of dipole
τ = P E sinθ d = separation between charge forming dipole.

In vector form $\vec{\tau} = \vec{P} \times \vec{E}$

- If θ is 0°, then τ= 0. If the dipole is placed along the field, then torque on it is zero. It is called stable equilibrium.
- If θ is 180°, then τ = 0. If the dipole is place anti parallel to the field, then torque on it is zero. It is called un stable equilibrium.
- If θ is 90°, then τ = PE. If the dipole is placed right angle to the field, then torque on it is maximum.
- When dipole is placed in uniform electric field, net force on it is always zero.
- If the dipole is place in uniform electric field, net force on it is non-zero but there is torque on it.
- If dipole is placed parallel or anti parallel to field, net torque on it zero, but net force on it is non- zero.
- If dipole moment of dipole is parallel to electric field, then net force on dipole is in the direction increasing field.
- If dipole moment of dipole is anti parallel to electric field, then net force in dipole is in the direction of decreasing field.

Continuous charge distribution.

It is the system of closely spaced charges. There are three kinds of continuous charge distributions. They are linear charge distribution, surface charge distribution and volume charge distribution.

Linear charge density : It is defined as the charge per unit length. It is denoted by λ or μ . Its unit is ***coulomb/meter or C/m***

If q charge is uniformly distributed over a line of length ℓ, then linear charge density is given by $\lambda = \frac{q}{\ell}$.
If the distribution of charges is not uniform, then we defined linear charge density at a point.

Surface charge density : It is defined as the charge per unit area. It is denoted by **σ**. Its unit is ***coulomb/meter² or C/m².***

If **q** charge is uniformly distributed over a surface **S**, then surface charge density is given by $\sigma = \frac{q}{s}$.

If the distribution of charges is not uniform, then we defined surface charge density at a point.

Volume charge density : It is defined as the charge per unit volume. It is denoted by p. Its unit is ***coulomb/meter³ of C/m³.***

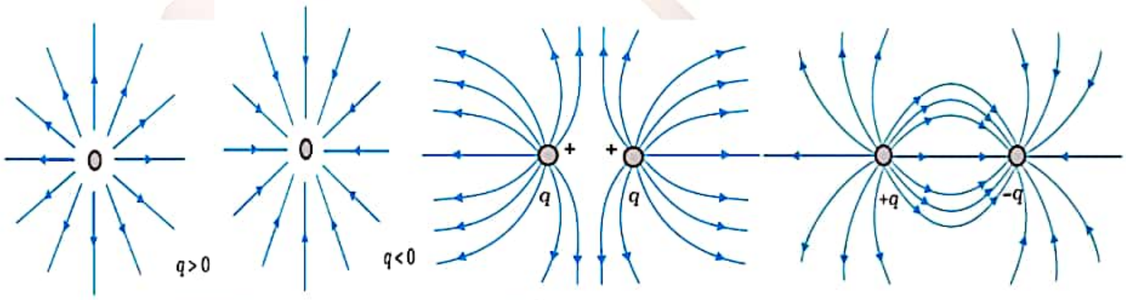
If q charge is uniformly distributed over a volume v, then volume charge density is given by $\mathbf{p} = \frac{q}{v}$.
If the distribution of charges is not uniform, then we define volume charge density at a point.

Electric lines of forces

Lines of force of field are the pictorial representation of electric field. Electric lines of force is a path along which unit positive charge tend to move in an electric field.

or

An imaginary line drawn along the unit positive charge.



Properties of electric lines of forces:

1) Electric lines of force start from positive charge and end on negative charge.(fig1)
2) Lines of forces are always normal to the surface of charged conductor (fig2)
3) Tangent dawn to the lines of force at any point gives the direction of electric field at that point.(fig3)
4) Lines of forces will never intersect: If they intersect, at the point of intersection we can draw two tangents. This means that there are two resultant directions along which unit positive charge placed at that point has to move simultaneously. This is not possible. Hence lines of force will never intersect each other. (fig4)

5) Lines of force do not pass through the conductor.
6) Lines of forces exert longitudinal tension which explains the attraction between the unlike charges.(fig5)
7) Lines of forces exert lateral pressure on one another which explains repulsion between like charges.(fig6)
8) Lines of forces are crowded at the region of stronger field and spread out at the region of weaker field. (fig7)
9) Equally spaced lines represent the uniform field where as unequal spaced lines represent the non uniform electric field.(fig8)

Note: Concept of line of force is imaginary and it has no physical significance.

Electric flux

Electric flux is the number of lines of force passing normally through any surface. Or The number of field lines crossing normal to the planar area element.

In general electric flux through an area element is $d\phi = E\ ds \cos\theta$ or $d\phi = EA \cos\theta$

Where **E** = Electric intensity

Ds = area of surface under consideration.

θ= angle between E and outward normal to area element ds.

If E is along the outward normal to area element ds, then $\theta = 0^{o}$ and $\cos 0 = 1$

$$\therefore d\phi = E\ ds$$

S.I unit of electric flux is – Webber.

To calculate the total flux through any given surface, divide the surface into small are elements. Now calculate the flux at each element and add them up. Hence total flux through entre surface is

Hence total electric flux through entire surface $\phi = \Sigma d\phi = \Sigma E\ ds \cos\theta$

Note : Area element vector **ds** at a point on a closed surface is $ds\hat{n}$. Where **ds**is the magnitude of area element and $\hat{n}$ is the unit vector in the direction of outward normal at that point.

State and Prove Gauss theorem.

Statement:*Total electric flux through any closed surface enclosing charge is equal to $1/\varepsilon_0$ times the net charge enclosed by the surface.*

Proof : Consider total flux through a sphere of radius r, which encloses point charge q at its centre. Divide the sphere into small area elements as in diagram.

The unit vector $\hat{r}$ is along the radius vector from centre to area element. Since normal to sphere at every point is along the radius vector at the point, the area element $\overrightarrow{ds}$and $\hat{r}$ have same direction.

$$\Phi = \vec{E}.\overrightarrow{ds} = E\ ds \cos\theta.\ \theta = 0.$$

Total flux through an area element ds is $d\Phi = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}ds$ **(Where $\hat{r}$= 1)**

Total flux through sphere is $\phi = \Sigma\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}ds$

Since each area element of sphere is at the same distance from charge

$$\phi = \sum \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}ds$$

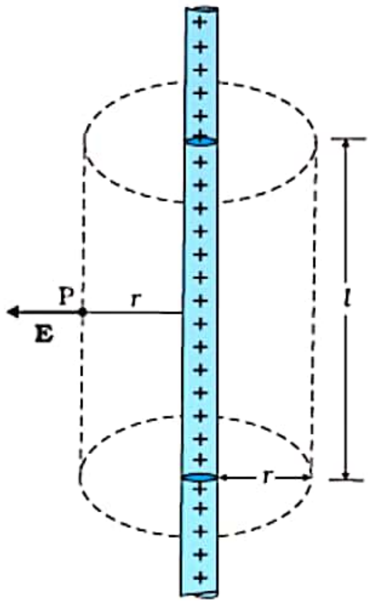
But $\sum ds = 4\pi\, r^2$ = Surface area of sphere

$$\phi = \sum \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}4\pi r^2$$

$$\phi = \frac{q}{\varepsilon_0}$$

- Gauss theorem is true for any closed surface, no matter what is shape or size.
- The term q includes the sum of all charges enclosed by the surface. The charges may be located anywhere inside the surface.
- The surface that we choose for the application of Gauss theorem is called the Gaussian surface.

‡ **10.** ***Obtain expression for Electric field due to an infinitely long straight uniformly charged wire using Gauss theorem.***



Consider an infinitely long thin straight wire of uniform linear charge densityλ. The wire has symmetry about an axis. Since the wire is infinite, electric field does not depend on position of the point along the length of the wire. The electric field is everywhere radial in the plane cutting the wire normally. The strength of electric field depends only on the radial distance **r.**

To calculate the field, imagine a cylindrical Gaussian surface, as shown in diagram. Since the field is everywhere radial, flux through the two ends of the cylindrical Gaussian surface is zero. At the cylindrical part of the surface. Electric field is normal to the surface at every point.

According to Gauss theorem

Total electric flux through any closed surface enclosing charge = $\frac{1}{\varepsilon_0}$ times the net charge enclosed by the surface.

WKT electric flux is given by $d\phi = E\,A\cos\theta$

$$d\phi = EA \quad (\theta = 0)$$

$$d\phi = E\times 2\pi r\ell \quad (A = 2\pi r\ell) \;\ldots\ldots\ldots.\; (1)$$

From gauss's law

$$d\phi = \frac{q}{\varepsilon_0} \;\ldots\ldots\ldots\ldots\; (2)$$

$$[\lambda = \frac{q}{\ell} \text{ or } q = \lambda\ell]$$

$$E 2\pi r\ell = \frac{\lambda\ell}{\varepsilon_0}$$

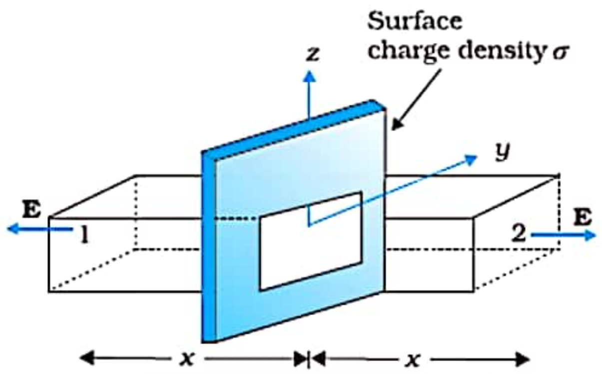
$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

In vector form

$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{n}$$

Where $\hat{n}$ is the radial unit vector in the plane normal to the wire and passing through the point.

‡ 11. Obtained expression for Electric field due to uniformly charged infinite plane sheet using Gauss theorem



Consider an infinite plane sheet of uniform surface charge density **σ**. By symmetry electric field at every point must be parallel to x-direction.

Imagine Gaussian surface in the form of rectangular parallelepiped of area of cross section **A**. As shown in diagram, only **1**and **2** faces of the parallelepiped contributes to the flux. Other face do not contribute to flux since electric field lines are parallel to their surfaces. Unit vector normal to surface **1** is in **– x** direction and Unit vector normal to surface **2** is in **+ x** – direction. Hence total electric flux through the Gaussian surface is **2 E A.** The charge enclosed by Gaussian surface is **σA.**

Total electric flux through any closed surface enclosing charge = times the net charge enclosed by the surface.

$$2\,E\,A = \frac{\sigma A}{\varepsilon_0}$$

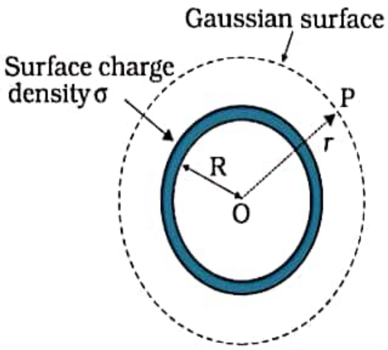
$$E = \frac{\sigma}{2\varepsilon_0}$$

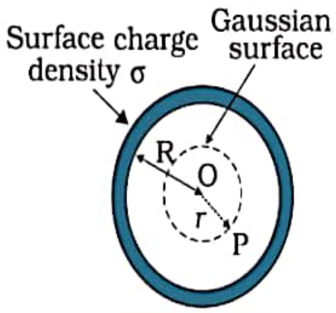
In vector form $\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n}$

Where $\hat{n}$ is the unit vector normal to the plane and going away from it.

‡ 12. Obtain expression for electric intensity at a point outside due uniformly charged thin spherical shell

Consider a spherical shell of radius r carrying +q. This charge distributes uniformly over its outer surface. Let P be a point at a distance d from the center of the spherical shell. Imagine a sphere with O as center and radius d. This imaginary spherical surface is called Gaussian surface. The point p lies on this surface.





The electric field is same at all points on the Gaussian surface and is along the normal to the Gaussian surface.

$\therefore \theta = 0^0$ and $\cos 0^0 = 1$

The according to Gauss theorem

$$\phi_E = \frac{q}{\varepsilon_0}$$

$$\sum E\, ds \cos\theta = \frac{q}{\varepsilon_0}$$

$$E \sum ds = \frac{q}{\varepsilon_0}$$

Where $\sum ds = 4\pi r^2$= surface are of Gaussian surface.

$$E.\, 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

In vector form $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{n}$

Where $\hat{n}$ *is the unit vector in the direction of radius vector OP.*

When the point is inside the conductor

Consider a spherical shell of radius **r** carrying charge **+q.** This charge distributes uniformly over its outer surface. Let **p** be a point inside at a distance **d** from the center of the spherical shell. Imagine a sphere with **O** as center and radius **d**. This imaginary spherical surface is called Gaussian surface. The point **p** lies on this surface.

The according to Gauss theorem

$$\phi_E = \frac{q}{\varepsilon_0}$$

$$\sum E\ ds\ \cos\theta = \frac{q}{\varepsilon_0}$$

In this case the Gaussian surface does not enclose any charge and q = 0

$$\therefore \sum E\ ds\ \cos\theta = 0$$

$$E = 0$$

Hence electric intensity inside the charge conductor is zero.

Physical quantity	Symbol	Dimensions	Unit	Remarks
Vector area element	$\Delta \mathbf{S}$	$[L^2]$	m^2	$\Delta \mathbf{S} = \Delta S\ \hat{\mathbf{n}}$
Electric field	**E**	$[MLT^{-3}A^{-1}]$	$V\ m^{-1}$	
Electric flux	ϕ	$[ML^3\ T^{-3}A^{-1}]$	V m	$\Delta\phi = \mathbf{E}.\Delta\mathbf{S}$
Dipole moment	**p**	[LTA]	C m	Vector directed from negative to positive charge
Charge density				
linear	λ	$[L^{-1}\ TA]$	$C\ m^{-1}$	Charge/length
surface	σ	$[L^{-2}\ TA]$	$C\ m^{-2}$	Charge/area
volume	ρ	$[L^{-3}\ TA]$	$C\ m^{-3}$	Charge/volume