Exercise 3.1

1. In the matrix A

$$\begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & \frac{5}{2} & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$$

Write
(i) The order of the matrix, (ii) The number of elements,
(iii) Write the elements a_{13}, a_{21}, a_{33}, a_{24}, a_{23}.

Solution:
(i) In given matrix,
Number of rows = 3
Number of column = 4
Therefore, Order of the matrix is 3 x 4.

(ii) The number of elements in the matrix A is 3 x 4 = 12.

(iii) a_{13} = element in first row and third column = 19

a_{21} = element in second row and first column = 35

a_{33} = element in third row and third column =

a_{24} = element in second row and fourth column = 12

a_{23} = element in second row and third column = 5/2

2. If a matrix has 24 elements, what are the possible orders it can have? What, if it has 13 elements?

Solution:
We know that, a matrix of order mxn having mn elements.

There are 8 possible matrices having 24 elements of orders are as follows:
1 x 24, 2 x 12, 3 x 8, 4 x 6, 24 x 1, 12 x 2, 8 x 3, 6 x 4.

Prime number 13 = 1 x 13 and 13 x 1

Again,1 x 13(Row matrix) and 13 x 1(Column matrix) are 2 possible matrices whose product is 13.

3. If a matrix has 18 elements, what are the possible orders it can have? What, if it has 5 elements?

Solution:

We know that, a matrix of order mxn having mn elements.

There are 6 possible matrices having 18 elements of orders:
1 x 18, 2 x 9, 3 x 6, 18 x 1, 9 x 2, 6 x 3.

Again, the product of 1 and 5 or 5 and 1 is 5.

Therefore, 1 x 5 (Row matrix) and 5 x 1 (Column matrix) are 2 possible matrices.

4. Construct a 2 × 2 matrix, A = [a_{ij}], whose elements are given by:

(i) $a_{ij} = \frac{(i+j)^2}{2}$

(ii) $a_{ij} = \frac{i}{j}$

(iii) $a_{ij} = \frac{(i+2j)^2}{2}$

Solution:

(i) Construct 2x2 matrix for

$$a_{ij} = \frac{(i+j)^2}{2}$$

Elements for 2x2 matrix are: $a_{11}, a_{12}, a_{21}, a_{22}$

For a_{11}, i = 1 and j = 1

$$a_{11} = \frac{1}{1} = 1$$

For a_{12}, i = 1 and j = 2

$$a_{12} = \frac{1}{2}$$

For a_{21}, i = 2 and j = 1

$$a_{21} = \frac{2}{1} = 2$$

For a_{22}, i = 2 and j = 2

$$a_{22} = \frac{2}{2} = 1$$

The required matrix is

$$\begin{bmatrix} 1 & 1/2 \\ 2 & 1 \end{bmatrix}$$

(iii) Construct 2x2 matrix for

$$a_{ij} = \frac{(i+2j)^2}{2}$$

Elements for 2x2 matrix are: $a_{11}, a_{12}, a_{21}, a_{22}$

For a_{11}, i = 1 and j = 1

$$a_{11} = \frac{(1+2)^2}{2} = \frac{(3)^2}{2} = \frac{9}{2}$$

For a_{12}, i = 1 and j = 2

For a_{11}, i = 1 and j = 1

$$a_{11} = \frac{(1+1)^2}{2} = \frac{(2)^2}{2} = \frac{4}{2} = 2$$

For a_{12}, i = 1 and j = 2

$$a_{12} = \frac{(1+2)^2}{2} = \frac{(3)^2}{2} = \frac{9}{2}$$

For a_{21}, i = 2 and j = 1

$$a_{21} = \frac{(2+1)^2}{2} = \frac{(3)^2}{2} = \frac{9}{2}$$

For a_{22}, i = 2 and j = 2

$$a_{22} = \frac{(2+2)^2}{2} = \frac{(4)^2}{2} = \frac{16}{2} = 8$$

Required matrix is :

$$\begin{bmatrix} 2 & 9/2 \\ 9/2 & 8 \end{bmatrix}$$

(ii) Construct 2x2 matrix for

$$a_{ij} = \frac{i}{j}$$

Elements for 2x2 matrix are: $a_{11}, a_{12}, a_{21}, a_{22}$

$$a_{12} = \frac{(1+4)^2}{2} = \frac{(5)^2}{2} = \frac{25}{2}$$

For a_{21}, i = 2 and j = 1

$$a_{21} = \frac{(2+2)^2}{2} = \frac{(4)^2}{2} = \frac{16}{2} = 8$$

For a_{22}, i = 2 and j = 2

$$a_{22} = \frac{(2+4)^2}{2} = \frac{(6)^2}{2} = \frac{36}{2} = 18$$

The required matrix is

$$\begin{bmatrix} 9/2 & 25/2 \\ 8 & 18 \end{bmatrix}$$

5. Construct a 3 x 4 matrix, whose elements are given by:

(i) $a_{ij} = \frac{1}{2}|-3i + j|$

(ii) $a_{ij} = 2i - j$

Solution:

(i) Construct 3 x 4 matrix for

$$a_{ij} = \frac{1}{2}|-3i + j|$$

Elements for 3 x 4 matrix are: $a_{11}, a_{12}, a_{13}, a_{14}, a_{21}, a_{22}, a_{23}, a_{24}, a_{31}, a_{32}, a_{33}, a_{34}$

For a_{11}, i = 1 and j = 1

$$a_{11} = \frac{1}{2}|-3+1| = \frac{1}{2}|-2| = \frac{1}{2}(2) = 1$$

For a_{12}, i = 1 and j = 2

$$a_{12} = \frac{1}{2}|-3+2| = \frac{1}{2}|-1| = \frac{1}{2}(1) = \frac{1}{2}$$

For a_{13}, i = 1 and j = 3

$$a_{13} = \frac{1}{2}|-3+3| = \frac{1}{2}|0| = \frac{1}{2}(0) = 0$$

For a_{14}, i = 1 and j = 4

$$a_{14} = \frac{1}{2}|-3+4| = \frac{1}{2}|1| = \frac{1}{2}(1) = \frac{1}{2}$$

For a_{21}, i = 2 and j = 1

$$a_{21} = \frac{1}{2}|-6+1| = \frac{1}{2}|-5| = \frac{1}{2}(5) = \frac{5}{2}$$

For a_{22}, i = 2 and j = 2

$$a_{22} = \frac{1}{2}|-6+2| = \frac{1}{2}|-4| = \frac{1}{2}(4) = 2$$

For a_{23}, i = 2 and j = 3

$$a_{23} = \frac{1}{2}|-6+3| = \frac{1}{2}|-3| = \frac{1}{2}(3) = \frac{3}{2}$$

For a_{24}, i = 2 and j = 4

$$a_{24} = \frac{1}{2}|-6+4| = \frac{1}{2}|-2| = \frac{1}{2}(2) = 1$$

For a_{31}, i = 3 and j = 1

$a_{31} = \frac{1}{2}|-9+1| = \frac{1}{2}|-8| = \frac{1}{2}(8) = 4$

For a32, i = 3 and j = 2

$a_{32} = \frac{1}{2}|-9+2| = \frac{1}{2}|-7| = \frac{1}{2}(7) = \frac{7}{2}$

For a33, i = 3 and j = 3

$a_{33} = \frac{1}{2}|-9+3| = \frac{1}{2}|-6| = \frac{1}{2}(6) = 3$

For a34, i = 3 and j = 4

$a_{34} = \frac{1}{2}|-9+4| = \frac{1}{2}|-5| = \frac{1}{2}(5) = \frac{5}{2}$

The required matrix is

$$\begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}$$

(ii) Construct 3 x 4 matrix for

$a_{ij} = 2i - j$

Elements for 3 x 4 matrix are: $a_{11}, a_{12}, a_{13}, a_{14}, a_{21}, a_{22}, a_{23}, a_{24}, a_{31}, a_{32}, a_{33}, a_{34}$

For a11, i = 1 and j = 1

$a_{11} = 2 - 1 = 1$

For a12, i = 1 and j = 2

$a_{12} = 2 - 2 = 0$

For a_{13}, i = 1 and j = 3

$a_{13} = 2 - 3 = -1$

For a_{14}, i = 1 and j = 4

$a_{14} = 2 - 4 = -2$

For a_{21}, i = 2 and j = 1

$a_{21} = 4 - 3 = 3$

For a_{22}, i = 2 and j = 2

$a_{22} = 4 - 2 = 2$

For a_{23}, i = 2 and j = 3

$a_{23} = 4 - 3 = 1$

For a_{24}, i = 2 and j = 4

$a_{24} = 4 - 4 = 0$

For a_{31}, i = 3 and j = 1

$a_{31} = 6 - 1 = 5$

For a_{32}, i = 3 and j = 2

$a_{32} = 6 - 2 = 4$

For a_{33}, i = 3 and j = 3

$a_{33} = 6 - 3 = 3$

For a_{34}, i = 3 and j = 4

$a_{34} = 6 - 4 = 2$

The required matrix is

$$\begin{bmatrix} 1 & 0 & -1 & -2 \\ 3 & 2 & 1 & 0 \\ 5 & 4 & 3 & 2 \end{bmatrix}$$

6. Find the values of x, y and z from the following equations:

(i) $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$

(ii) $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

(iii) $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

Solution:

(i)

$$\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$$

Since both the matrices are equal, so their corresponding elements are also equal.

Find the value of unknowns by equating the corresponding elements.

4 = y
3 = z
x = 1

(ii) Since both the matrices are equal, so their corresponding elements are also equal.

Find the value of unknowns by equating the corresponding elements.

$$\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$$

x+y = 6 ...(1)

5 + z = 5 => z = 0

xy = 8 ....(2)

From equation (1), x = 6 – y

Substitute the value of x in equation (2)

(6 – y)y = 8

6y – y^2 = 8

or y^2 – 6y + 8 = 0

(y – 4)(y – 2) = 0

y = 4 or y = 2

Put values of y in equation (1), x+y = 6 , we have x = 2 and x = 4

Therefore, x = 2, y = 4 and z = 0.

(iii)

Since both the matrices are equal, so their corresponding elements are also equal.

Find the value of unknowns by equating the corresponding elements.

$$\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$$

x + y + z = 9 ...(1)

x + z = 5 ...(2)

y + z = 7 ...(3)

equation (1) – equation (2), we get

y = 4

Equation (3): 4 + z = 7 => z = 3

Equation (2) : x + 3 = 5 => x = 2

Answer: x = 2, y = 4 and z = 3

7. Find the value of a, b, c and d from the equation:

$$\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$

Solution:

Equate the corresponding elements of the matrices:

a - b = -1 ...(1)

2a + c = 5 ..(2)

2a – b = 0 ...(3)

3c + d = 13 ...(4)

Equation (1) - Equation (3)

-a = -1 => a = 1

Equation (1) => 1 - b = -1 => b = 2

Equation (2) => 2(1) + c = 5 = > c = 3

Equation (4) => 3(3) + d = 13 => d = 4

Therefore, a = 1, b = 2, c = 3 and d = 4

8. A = $[a_{ij}]_{m \times n}$ is a square matrix, if

(A) m < n (B) m > n (C) m = n (D) None of these

Solution:

Option (C) is correct.

According to square matrix definition: Number of rows = number of columns (m = n)

9. Which of the given values of x and y make the following pair of matrices equal

$$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$$

(A) $x = \frac{-1}{3}$, $y = 7$ (B) Not possible to find

(C) $y = 7$, $x = \frac{-2}{3}$ (D) $x = \frac{-1}{3}$, $y = \frac{-2}{3}$

Solution:

Option (B) is correct.

Explanation:

By equating all corresponding elements, we get

3x + 7 =0 = > x = -7/3

y – 2 = 5 = > y = 7

y + 1 = 8 => y = 7

2 – 3x = 4 => x =-2/3

10. The number of all possible matrices of order 3 × 3 with each entry 0 or 1 is:

(A) 27 (B) 18 (C) 81 (D) 512
Solution:

Option (D) is correct.

The number of elements of 3x3 matrix is 9.

First element, a_11 is 2, can be 0 or 1, similarly the number of choices for each other element is 2.

Total possible arrangements = 2^9 = 512