

### Exercise 3.3

1. Find the transpose of each of the following matrices:

(i)  $\begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}$       (ii)  $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

(iii)  $\begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$

**Solution:**

(i) Let  $A = \begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}$ , Then  $A' = \begin{bmatrix} 5 & \frac{1}{2} & -1 \end{bmatrix}$

(ii) Let  $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ , Then  $A' = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

(iii) Let  $A = \begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$ , Then  $A' = \begin{bmatrix} -1 & \sqrt{3} & 2 \\ 5 & 5 & 3 \\ 6 & 6 & -1 \end{bmatrix}$

2. If  $A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$  and  $B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$  then verify that:

(i)  $(A - B)' = A' - B'$       (ii)  $(A - B)' = A' - B'$

**Solution:**

(i)  $A + B = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 3 & -2 \\ 6 & 9 & 9 \\ -1 & 4 & 2 \end{bmatrix}$

$$\text{Now, } (A+B)' = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix}$$

Again,

$$\begin{aligned} A' + B' &= \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ -2 & 1 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix} \end{aligned}$$

Proved.

$$(ii) \quad A - B = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 8 \\ 4 & 5 & 9 \\ -3 & -2 & 0 \end{bmatrix}$$

$$(A - B)' = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix}$$

$$\begin{aligned} A' - B' &= \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ -2 & 1 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix} \end{aligned}$$

Hence proved.

$$3. \text{ If } A' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} \text{ then verify that:}$$

$$(i) (A - B)' = A' - B'$$

$$(ii) (A - B)' = A' - B'$$

**Solution:**

$$A' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

As we know that  $(A')' = A$ , we have

$$A = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$$

$$\begin{aligned} A + B &= \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 1 & 1 \\ 5 & 4 & 4 \end{bmatrix} \end{aligned}$$

LHS:

$$(A + B)' = \begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix}$$

RHS:

$$A' + B' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix}$$

LHS = RHS

(ii)

$$A - B = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 4 & -3 & -1 \\ 3 & 0 & -2 \end{bmatrix}$$

LHS

$$(A-B)' = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$$

RHS:

$$A' - B' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$$

LHS = RHS

4. If  $A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$  and  $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$  then find  $(A + 2B)'$ .

**Solution:**

$$(A')' = A = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix}$$

Find  $A + 2B$

$$= \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} + 2 \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -4 & 1 \\ 5 & 6 \end{bmatrix}$$

And

$$(A+2B)' = \begin{bmatrix} -4 & 5 \\ 1 & 6 \end{bmatrix}$$

5. For the matrices  $A$  and  $B$ , verify that  $(AB)' = B'A'$ , where:

$$(i) \quad A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}, \quad B = [-1 \quad 2 \quad 1]$$

$$(ii) \quad A = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \quad B = [1 \quad 5 \quad 7]$$

**Solution:** LHS

$$AB = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix} \begin{bmatrix} -1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}$$

$$(AB)' = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$$

RHS:

$$B'A' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & -4 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$$

LHS = RHS

(ii)

$$AB = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 5 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 5 & 7 \\ 2 & 10 & 14 \end{bmatrix}$$

LHS

$$(AB)' = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$$

RHS:

$$B'A' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$$

LHS = RHS

**6.**

(i) If  $A =$  then verify that  $A'A = I$ .

(ii) If  $A =$  then verify that  $A'A = I$ .

**Solution:**

$$\begin{aligned} \text{L.H.S.} = \mathbf{AA}' &= \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \\ &= \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \\ &= \begin{bmatrix} \cos^2 \alpha + \sin^2 \alpha & \cos \alpha \sin \alpha - \sin \alpha \cos \alpha \\ \sin \alpha \cos \alpha - \cos \alpha \sin \alpha & \sin^2 \alpha + \cos^2 \alpha \end{bmatrix} \end{aligned}$$

$$(i) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \mathbf{I} = \text{R.H.S.}$$

(ii)

$$\begin{aligned} \text{L.H.S.} = \mathbf{A}'\mathbf{A} &= \begin{bmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{bmatrix} \begin{bmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{bmatrix} \\ &= \begin{bmatrix} \sin^2 \alpha + \cos^2 \alpha & \sin \alpha \cos \alpha - \cos \alpha \sin \alpha \\ \cos \alpha \sin \alpha - \sin \alpha \cos \alpha & \cos^2 \alpha + \sin^2 \alpha \end{bmatrix} \end{aligned}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \mathbf{I} = \text{R.H.S.}$$

7. (i) Show that the matrix  $\mathbf{A} = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$  is a symmetric matrix.

(ii) Show that the matrix  $\mathbf{A} = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$  is a skew symmetric matrix.

**Solution:**

According to the symmetric matrix definition:  $\mathbf{A}' = \mathbf{A}$

$$\mathbf{A}' = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix} = \mathbf{A}$$

$\mathbf{A}$  is a symmetric matrix.

(ii) According to the skew symmetric matrix definition:  $A' = -A$

$$\begin{aligned} A' &= \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} \\ &= - \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} \\ &= -A \end{aligned}$$

A is a skew symmetric matrix.

8. For a matrix  $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$ , verify that:

(i)  $(A + A')$  is a symmetric matrix.

(ii)  $(A - A')$  is a skew symmetric matrix.

**Solution:**

(i)

$$A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$$

$$A + A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}$$

$$(A + A')' = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix} = A + A'$$

$(A + A')$  is a symmetric matrix.

(ii)

$$A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$$

$$A - A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$(A+A')' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = -(A + A')$$

$(A - A')$  is a skew symmetric matrix.

**9. Find  $\frac{1}{2}(A + A')$  and  $\frac{1}{2}(A - A')$  when  $A$  is**

$$\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

**Solution:**

$$A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} \text{ then } A' = \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$$

Now,  $A + A'$  is

$$\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} + \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So,  $\frac{1}{2}(A + A')$  is

$$\frac{1}{2} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Again ,

$$A - A' = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} - \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2a & 2b \\ -2a & 0 & 2c \\ -2b & -2c & 0 \end{bmatrix}$$

$$\frac{1}{2}(A - A') = \frac{1}{2} \begin{bmatrix} 0 & 2a & 2b \\ -2a & 0 & 2c \\ -2b & -2c & 0 \end{bmatrix} = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

**10. Express the following matrices as the sum of a symmetric and skew symmetric matrix:**

(i)  $\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$       (ii)  $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

(iii)  $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$       (iv)  $\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$

**Solution:**

(i) Let  $A = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$  then,  $A' = \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$

Symmetric matrix =  $\frac{1}{2}(A + A')$

$$= \frac{1}{2} \left( \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix}$$

And Skew symmetric matrix =  $\frac{1}{2}(A - A')$

$$= \frac{1}{2} \left( \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

Again,

Symmetric matrix + Skew symmetric matrix =

$$\begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$$

Which is A.

Given matrix is sum of Symmetric matrix and Skew symmetric matrix .

$$(ii) \text{ Let } A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \text{ then, } A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

Symmetric matrix =  $1/2 (A + A')$

$$= \frac{1}{2} \left( \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

And Skew symmetric matrix =  $1/2(A - A')$

$$= \frac{1}{2} \left( \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} - \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Symmetric matrix + Skew symmetric matrix =

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

Which is A.

Given matrix is sum of Symmetric matrix and Skew symmetric matrix .

(iii)

$$\text{Let } A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} \text{ then, } A' = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

Symmetric matrix =  $1/2 (A + A')$

$$= \frac{1}{2} \left( \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix}$$

And Skew symmetric matrix =  $1/2(A - A')$

$$= \frac{1}{2} \left( \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ -\frac{5}{2} & 0 & 3 \\ -\frac{3}{2} & -3 & 0 \end{bmatrix}$$

Symmetric matrix + Skew symmetric matrix =

$$\begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ -\frac{5}{2} & 0 & 3 \\ -\frac{3}{2} & -3 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$$

Which is A.

Given matrix is sum of Symmetric matrix and Skew symmetric matrix .

(iv) Let  $A = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$  then,  $A' = \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}$

Symmetric matrix =  $\frac{1}{2}(A + A')$

$$\frac{1}{2} \left( \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix} \right) = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$$

And Skew symmetric matrix =  $\frac{1}{2}(A - A')$

$$\frac{1}{2} \left( \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix} \right) = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$$

Symmetric matrix + Skew symmetric matrix =

$$\begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$$

Which is A.

Given matrix is sum of Symmetric matrix and Skew symmetric matrix .

Choose the correct answer in Exercises 11 and 12.

11. If A and B are symmetric matrices of same order,  $AB - BA$  is a:

(A) Skew-symmetric matrix

(B) Symmetric matrix

(C) Zero matrix

(S) Identity matrix

**Solution:**

Option (A) is correct.

12.

If  $A = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$ , and  $A + A' = I$ , then the value of  $\alpha$  is

(A)  $\frac{\pi}{6}$

(B)  $\frac{\pi}{3}$

(C)  $\pi$

(D)  $\frac{3\pi}{2}$

**Solution:**

Option (B) is correct.

$$\begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} + \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2\cos \alpha & 0 \\ 0 & 2\cos \alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By equating corresponding terms,

$$2\cos \alpha = 1$$

$$\cos \alpha = \cos \frac{\pi}{3}$$

$$\alpha = \frac{\pi}{3}$$